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**RANDOM LOADING FATIGUE CRACK GROWTH  
— CRACK CLOSURE CONSIDERATIONS**

Final Report

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## ABSTRACT

The prediction of fatigue crack growth is an important element of effective fracture control for metallic structures and mechanical components, especially in the aerospace industry. The prediction techniques available and applied today are mostly based on fatigue crack growth measurements determined in constant amplitude testing. However, while many service loadings are constant amplitude, many more loadings are random amplitude. This report is concerned with an investigation to determine which statistics of random loadings are relevant to fatigue crack closure. This report briefly reviews the fundamentals of random processes and crack closure, then it qualitatively discusses the relevance of certain random process parameters to the crack closure calculation. A course for further research is outlined.

## 1. INTRODUCTION

The growth of fatigue cracks to fracture is the most critical life-limiting criterion for almost all metallic structures and mechanical components. This is especially true of aerospace structures and components because of the high stresses they experience. The risk posed by fatigue and fracture is significant and its management is known as fracture control. Two elements essential to effective fracture control are the control of the initial sizes of cracks, flaws, or crack-like defects, and the prediction of the growth of fatigue cracks.

Initial crack sizes are typically controlled by manufacturing procedures and inspections. For example, to make a fastener hole one might either drill the hole directly, or first drill a smaller pilot hole and then ream it. The initial flaw sizes characteristic of the two processes might be expected to be different. The fastener holes would then be subjected to nondestructive inspection. There are several techniques commonly used, each has limitations on the size flaw that can be detected with confidence. Together the initial flaw size distribution and the flaw detectability define the probability distribution of flaw sizes that go into service.

For fracture control purposes, the largest initial flaw size expected to pass through manufacturing and inspection into service is chosen as the starting point for fatigue crack growth calculations. In order to facilitate accurate and consistent calculations, NASA has developed and distributed a computer code called FLAGRO [Forman et al., 1986]. This program automatically computes the increments of crack growth for the specified loading, and continues the calculations until the crack reaches its final critical size and shape. The fatigue loading is specified by a "loading spectrum," which is a tabulation of the number of stress cycles occurring in different stress ranges.

Standard loading spectra have been developed for various applications, for example, the NASA Goddard Space Flight Center's spectrum developed for payloads carried by the Space Shuttle. The growth increment for a given stress cycle is determined by "linear damage accumulation theory," which means that any effects related to potential interactions between fatiguing loads are ignored.

Load interaction effects have been extensively documented for overloads introduced into constant amplitude crack growth tests. It is observed that a large tensile overload will cause a reduction in the crack growth rate for some time after the overload. This behavior is accounted for by crack closure theory which states that an overload leaves behind residual stresses which hold the crack closed for a portion of the subsequent stress cycles. The effective amplitude of the subsequent stress cycles and the corresponding growth rate are thereby reduced. Hence, in some loadings load interaction effects may be quite important.

Many fatigue loadings, especially in the aerospace industry, are actually random processes for which the interaction effects are probably significant. So it is

not surprising that members of industry have recently asked NASA to add load interaction effects to FLAGRO's capabilities. This capability should make the crack growth predictions more accurate. Furthermore, since the dominant effect of load interactions is to prolong fatigue life, this would have a beneficial impact on future designs via weight reductions. It would also enable the certification of existing structures for longer life extensions, which is emerging as a critical issue for NASA on both the Space Shuttle and the Space Station.

Two types of approaches to the random loading problem are found in the literature: cycle-by-cycle calculations and calibrations to characteristic stresses. In the cycle-by-cycle approach, crack growth is calculated one cycle at a time with the order of cycles preserved. The crack growth rate for each cycle is determined from constant amplitude fatigue tests, and load interactions are accounted for by reducing the effective stress range by a factor related to plastic zone size, e.g., Wheeler [reference] and Willenborg [reference].

In the characteristic stress approach, the crack growth rate under random loading is re-calibrated to a characteristic value of the stress intensity factor, typically the root-mean-square (RMS) value. The sequence of loads is lost, and a new crack growth rate curve must be generated for each new loading. The crack growth rate curve is typically reported as the growth per flight,  $da/dF$ , for example.

However, before industry adopts a "nonlinear damage accumulation theory", the impact on fracture control safety margins should be assessed so that the probability of fracture is not radically changed. The currently established scheme for fracture control is based on and calibrated to a long experience base. Certain probabilities of fracture are achieved by these practices and have been accepted implicitly by the engineering community. If one element of the scheme is perturbed, other elements must also be adjusted. For example, if one could predict longer fatigue lives with greater certainty, one might be able to tolerate larger initial flaws, but the accuracy of the inspection may have to be increased in order to maintain the same probability of fracture.

Therefore, the random loading fatigue problem is really two-fold. First, a better understanding of the mechanisms of fatigue crack growth under random loading is necessary so that more accurate crack growth predictions can be made. Second, a better understanding of the true safety margins (i.e., probabilities of fracture) achieved by current practices is necessary so that new practices may be confidently introduced. This research project is concerned only with the prediction aspect of the random loading problem. Understanding the safety margins would logically be a second project.

This report examines the interplay between random loadings and fatigue crack closure as a first step in defining a research program on random loading fatigue. Section 2 provides background on random loadings and crack closure. Section 3

examines the prediction of crack closure under known variable amplitude fatigue loadings. Section 4 then extrapolates the concepts to random loadings. Section 5 presents a course for future research.

## 2. BACKGROUND

This section briefly reviews the basic concepts of random loadings and fatigue crack closure. Due to the limited space available, a comprehensive literature review is not attempted.

### 2.1 Random Loading Statistics

Figures 1a and 1b show samples of two different processes which might represent the fatigue loading on a crack. The theory of random processes is well known. We will only briefly headline the important concepts below. For a more in-depth treatment a number of standard texts are available, e.g., Bendat and Piersol [1986].

The probability that a random process  $X(t)$  will be found below a value  $x_0$  at an arbitrary time  $t_0$  is given by the marginal probability distribution function (PDF) of the process, denoted  $F_X(x_0)$ . The expected value or mean,  $E[X(t)] = \mu_X(t)$ , and the variance,  $\sigma_X^2(t)$ , are the most important statistics of the process and are generally time dependent. The square root of the variance is the standard deviation,  $\sigma_X(t)$ .

The normal (or Gaussian) distribution is commonly applied to random processes of engineering interest. It has two parameters, the mean,  $\mu$ , and the variance,  $\sigma^2$ . Its equation is

$$F_X(x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}\sigma} \exp \left[ -\frac{(t-\mu)^2}{2\sigma^2} \right] dt \quad (1)$$

A process with this PDF is called a Gaussian process.

The probability that, at two times  $t_1$  and  $t_2$ ,  $X(t)$  will be found below  $x_1$  and  $x_2$ , respectively, is given by the joint PDF,  $F_{X(t_1), X(t_2)}(x_1, x_2)$ . Higher order distributions may also be defined, but are not used very often. The joint PDF is used to calculate the autocorrelation function which is defined as the expected value of  $X(t_1)$  times  $X(t_2)$ , i.e.,  $R_X(t_1, t_2) = E[X(t_1)X(t_2)]$ .

If the joint PDF remains constant over all time, the process is called stationary. Otherwise, the process is nonstationary. For stationary processes, the mean is constant,  $\mu_X(t) = \mu_X$ , the variance is constant,  $\sigma_X^2(t) = \sigma_X^2$ , and the autocorrelation function depends only on the length of the time difference  $\tau = t_2 - t_1$ ,  $R_X(\tau) = E[X(t)X(t+\tau)]$ . Note that for  $\tau = 0$ , the autocorrelation function is equal to the mean-square value of the process,  $R_X(0) = \sigma_X^2 + \mu_X^2$ .

It is necessary to subtract the mean value of the process before any further analysis. The linearly transformed process then has a mean of zero. If necessary, the mean can always be added back. Then the mean-square value given by the autocorrelation function at  $\tau = 0$  is equal to the variance,  $R_X(0) = \sigma_X^2$ , and the square root of the mean-square value (RMS) is equal to the standard deviation,  $\sigma_X$ .

The random processes of greatest engineering significance are Gaussian, stationary, and have zero-means. Once one has obtained the autocorrelation of such a process all the important statistical properties can be derived from it. These processes are assumed for the remaining discussion unless otherwise indicated.

The spectral density function,  $S_X(\omega)$ , is the Fourier transform of the autocorrelation function, and thus contains the same statistical information. While the autocorrelation function has great mathematical significance, the spectral density function is more useful to engineers because it gives us the distribution of energy in a random process as a function of frequency,  $\omega$ . Figures 2a and 2b show the spectral density functions for the processes shown previously in Figures 1a and 1b, respectively. In Figure 2a it is easily seen that the energy of the first signal is concentrated in a very narrow band of frequencies. This is called a narrowband process and it manifests itself as a sine wave of slowly varying random amplitude and phase shift. In Figure 2b it is seen that the energy of the second signal is spread out over a wider band of frequencies. This is called a wideband process and it manifests itself as a highly irregular waveform, what is commonly thought of as "random".

From the properties of Fourier transforms, the area under the spectral density function is equal to the value of the autocorrelation function at  $\tau = 0$ , which is equal to the variance of a zero-mean process,  $\sigma_X^2$ . The square root of the area is thus the RMS of the process, equal to the standard deviation,  $\sigma_X$ .

The area under the spectrum is a special case of a moment of the spectral density function. More generally, the  $k$ -th moment of the spectrum is given by

$$m_k = \int_0^\infty |\omega|^k S_X(\omega) d\omega \quad (2)$$

The area is the zero-eth moment,  $k = 0$ . Other moments, particularly  $k = 2$  and  $k = 4$ , are used to calculate the rate at which the process crosses different levels (level crossings), such as,  $x = 0$  (zero-crossings), and the rate at which peaks occur in the process. The ratio of the rate of zero-crossings to the rate of peaking is known as the irregularity factor,  $\alpha$ , and is a measure of the bandwidth of the process,  $0 \leq \alpha \leq 1$ . Note that  $\alpha = 1$  is the ideal theoretical narrowband process. Also note that  $\epsilon = \sqrt{1 - \alpha^2}$  and  $k = 1/\alpha$  are also used as bandwidth measures.

The spectral moments are also used to determine the probability distribution of extrema, i.e., peaks and valleys. These distributions depend on  $\alpha$  and on  $\sigma_X^2$ .

The author has derived approximate distributions for the height of the rise from a valley to the next peak and for the amplitude of rainflow counted fatigue cycles which depend on  $\alpha$  and  $\sigma_X^2$  as well as on other spectral moments [Ortiz and Chen, 1987]. These distributions are useful for calculating linear fatigue damage.

Not all random processes of interest to engineers are stationary. In general, nonstationary processes are much more difficult to handle and must be treated on a case-by-case basis. Standard approaches to these problems exist, but are beyond the scope of this section.

## 2.2 Fatigue Crack Closure

The correlation between the rate of fatigue crack growth,  $da/dN$ , and the cyclic range of the elastic stress intensity factor,  $\Delta K = K_{max} - K_{min}$ , is well established. However, to understand crack closure, it is necessary to reconsider the evolution of the plastic strain field as the crack grows longer. We summarize below the key elements of the theory, first proposed by Elber [1971]. For an extensive review, see Banerjee [1984].

On loading the crack from  $K_{min}$  to  $K_{max}$ , the material at the crack tip undergoes monotonic plastic straining and the crack tip extends. The zone of plastically deformed material has a width of approximately

$$r_p \approx \begin{cases} \frac{1}{2\pi} \left( \frac{K_{max}}{\sigma_y} \right)^2 & \text{plane stress} \\ \frac{1}{6\pi} \left( \frac{K_{max}}{\sigma_y} \right)^2 & \text{plane strain} \end{cases} \quad (3)$$

Note that, since  $K_{max}$  typically depends on  $S_{max}\sqrt{\pi a}$ ,  $r_p$  depends linearly on  $a$  and quadratically on  $S_{max}$ . Figure 3 illustrates the extent of the monotonic plastic wake for constant amplitude fatigue loading.

When the crack is unloaded, the singularity at the crack tip continues to exist until the crack faces meet, i.e., until the crack is closed. The crack tip stress field reverses. However, due to the Bauschinger effect the effective stress intensity on unloading is only half as much as on loading. Thus, the reversed plastic zone is only one-fourth (i.e., one-half squared) the width of the monotonic plastic zone. Figure 3 also illustrates the extent of the reversed plastic zone.

One result of this plastic deformation is obviously crack extension. Another, less obvious, result is that there is net stretching of the plastic zone. This creates a compressive residual stress field in the plastic wake behind the crack tip. When the crack is unloaded, the crack faces meet before the load is fully removed, closing the crack. The stress at which this occurs is called the crack closing stress. On re-loading, the crack remains closed until sufficient load is applied to open the crack, the crack opening stress,  $S_{op}$  (or  $K_{op}$ ). Experimental evidence indicates that the

opening and closing stresses are approximately the same. The importance of crack closure to fatigue crack growth is that there is no plastic deformation at the crack tip until the crack opens. In effect,  $\Delta K$  is reduced to  $\Delta K_{eff} = K_{max} - K_{op}$ .

Various researchers have attempted to predict  $S_{op}$  for constant amplitude loadings. The most extensive study to date is by Newman [1984], who fit the following equations to finite element model results:

$$S_{op}/S_{max} = \begin{cases} A_0 + A_1 R + A_2 R^2 + A_3 R^3 & \text{for } R \geq 0 \\ A_0 + A_1 R & \text{for } -1 \leq R < 0 \end{cases} \quad (4)$$

where

$$A_0 = (0.825 - 0.34\alpha + 0.05\alpha^2)[\cos(\pi S_{max}/2\sigma_o)]^{1/\alpha} \quad (5)$$

$$A_1 = (0.415 - 0.071\alpha)S_{max}/\sigma_o \quad (6)$$

$$A_2 = 1 - A_0 - A_1 - A_3 \quad (7)$$

$$A_3 = 2A_0 + A_1 - 1 \quad (8)$$

The opening stress is seen to depend on  $S_{max}$ , the R-ratio,  $R = S_{min}/S_{max} = K_{min}/K_{max}$ , and  $\sigma_o$ , the material's flow stress, which Newman takes to be the average of the uniaxial yield stress and the uniaxial tensile strength, and on  $\alpha$ , a "constraint" factor on tensile yielding. The material is assumed to yield when the applied stress is equal to  $\alpha\sigma_o$ . For plane stress,  $\alpha = 1$ , and for plane strain,  $\alpha = 3$ .

Crack closure has been used to explain the dependence of  $da/dN$  on the R-ratio, which is approximated by Forman's law

$$da/dN = \frac{C\Delta K^m}{(1-R)K_c - \Delta K} \quad (9)$$

According to the crack closure theory,  $da/dN$  should correlate with the effective range of the stress intensity factor,  $\Delta K_{eff} = K_{max} - K_{op}$ , which is somewhat less than the nominal  $\Delta K = K_{max} - K_{min}$ . Limited experimental results show good agreement.

Crack closure theory may also be used to explain the load interaction effects which occur when stress ranges of different amplitudes are intermingled in a load history. According to the theory, an occasional tensile overload in an otherwise constant amplitude stress history creates a residual stress field which increases the crack opening stress over a portion of the crack path ahead of the crack tip. This reduces  $\Delta K_{eff}$  and the corresponding  $da/dN$  until the crack grows out of the affected region. This effect is known as "crack growth retardation". Similarly, an occasional compressive overload can reduce the opening stress and thereby increase  $da/dN$ . This is known as "crack growth acceleration".



The majority of theoretical and experimental work on crack closure has been done on constant amplitude loadings. In the following sections we discuss the prediction of crack closure under variable amplitude and random amplitude loadings.

### 3. CRACK OPENING STRESS FOR VARIABLE LOADING

The subject of crack closure under variable amplitude loadings is still very new. In the occasional overload problem, the crack opening stress obviously varies as the crack grows. In a complex variable amplitude loading, such as shown in Figure 4, the crack opening stress could vary significantly during the load history, but there are reasons why it may not. In this section we will try to understand why  $S_{op}$  might or might not vary significantly during a variable amplitude loading, and how it might be predicted.

#### 3.1 Direct Approach

Consider a variable amplitude loading such as shown in Figure 4. Suppose this same loading block is repeated over and over. And for the moment, suppose that the crack does not grow during a single application of the block. It is evident that the maximum extent of the monotonic plastic zone is determined by the highest stress peak,  $S_{maxh}$ . The net plastic stretching is also governed by the lowest stress valley,  $S_{minl}$ . Therefore, the crack opening stress should be a function of the R-ratio:

$$R_{lh} = S_{minl}/S_{maxh} \quad (10)$$

Now let us relax the restriction that the crack does not grow during a single application of the loading block. If the crack grows "slowly" during the block, the net plastic stretching and, hence, the crack opening stress will still be controlled by  $R_{lh}$ . By "slowly" we mean the crack tip should not grow out of the plastic zone during the block. The plastic wake behind the crack tip for the variable amplitude loading should appear to be smooth, and should be the same wake as for constant amplitude loading from  $S_{minl}$  to  $S_{maxh}$ . Figure 5 illustrates the concept and shows a counter-example for which the crack does grow out of the maximum plastic zone, as for example in the occasional overload problem.

Thus, if the crack is growing slowly,  $S_{op}$  will remain constant, the plastic wake will be the same as for constant amplitude loading from  $S_{minl}$  to  $S_{maxh}$ , and predictable using  $R_{lh}$  in the relationships found in constant amplitude tests. The crack growth rate can then be predicted from  $\Delta K_{eff}$ . The method described has been proposed by Schijve [1980]. The correlation of his predictions with experimental results appears promising, but is not perfect. The errors in Schijve's predictions are most probably due to using Elber's original equations for  $S_{op}$ . Perhaps Newman's more recent equations would give better results.

### 3.2 Iterative Approach

One potential problem with the above approach has to do with the following. In a variation of occasional overload testing, the overload is followed immediately by a second identical overload. The observed retardation is longer than for a single overload [Banerjee, 1984]. Since the plastic zone theoretically is not enlarged by a significant amount, this suggests that the second overload somehow strengthens the compressive residual stresses. One infers that the crack opening stress observed in constant amplitude tests is not necessarily the same as observed in occasional overload testing. So perhaps one should not use the highest peak and the lowest valley to define the equivalent constant amplitude R-ratio, but rather the second highest peak, or an average?

Newman [unpublished] suggests the following iterative procedure:

First iteration:

1. Find the highest peak,  $S_{maxh}$ , and the lowest valley,  $S_{minl}$ .
2. Calculate an initial  $S_{op}^*$  using  $S_{minl}$ ,  $S_{maxh}$  and

$$R^* = S_{minl}/S_{maxh} \quad (11)$$

3. For each stress cycle, calculate the initial effective stress ranges

$$\Delta S_i^* = S_{max,i} - S_{op}^* \quad (12)$$

Second iteration:

4. Calculate the weighted average stress range given by

$$\overline{\Delta S} = \left[ \frac{1}{N_T} \sum_{i=1}^{N_T} \Delta S_i^{*m} \right]^{1/m} \quad (13)$$

where  $m$  is Paris exponent and only "positive" cycles, totaling  $N_T$ , are counted.

5. Calculate the average effective maximum stress

$$\overline{S_{max}} = S_{op}^* + \overline{\Delta S} \quad (14)$$

6. Calculate the final effective opening stress  $\overline{S_{op}}$  using the lowest valley  $S_{minl}$ , the average effective maximum  $\overline{S_{max}}$  and the final effective R-ratio

$$\overline{R} = S_{minl}/\overline{S_{max}} \quad (15)$$

7. Calculate the final effective stress ranges with which to calculate  $\Delta K_{eff}$

$$\Delta S_{eff,i} = S_{max,i} - \overline{S_{op}} \quad (16)$$

Note that the first three steps correspond to Schijve's method. The next three steps represent a second iteration to get an average effective R-ratio in step 6. Step 7 is then the calculation of the effective  $\Delta K$ .

### 3.3 Discussion

At this point it is impossible to say which of the two methods presented is better. Again, the rate at which the crack grows through the plastic zone of the "overload" is of great importance. If the crack is growing very slowly, it might take several or many blocks to get through the plastic zone. There could be a reinforcing of the plastic zone by repeated applications of the loading block. In this case the plastic wake would be similar to the constant amplitude loading wake. Schijve's approach would appear to be reasonable under these circumstances.

On the other hand, if the crack is growing so quickly or if the loading block is so long that the crack tip advances significantly during a block, there might not be any reinforcing of the plastic zone. In this case the plastic wake would not be similar to the constant amplitude loading wake. In fact, the wake would be irregular and  $S_{op}$  would vary non-monotonically. In this case, Newman's approach might be an acceptable approximation. The conditions under which either method would be valid must be established by experiment.

So far we have assumed that the same variable amplitude block is repeated over and over again. However, in real loadings each variable amplitude block would be different in many respects. In fact, the loading blocks would likely be random. This leads us to extend the discussion to random amplitude loadings in the next section.

## 4. CRACK OPENING STRESS FOR RANDOM LOADING

From our discussion so far it is evident that the crack opening stress,  $S_{op}$ , for a random amplitude loading will probably vary randomly. That is,  $S_{op}$  is also a random process. For a stationary random loading,  $S_{op}$  should also be stationary. For a nonstationary random loading,  $S_{op}$  could still be stationary because it should vary more slowly than the loading.

For the purpose of discussion, we first assume that the stress process,  $X(t)$ , is stationary. Assuming the process is Gaussian, which is often the case, the mean value,  $\mu_X$ , and the variance,  $\sigma_X^2$ , define the marginal probability distribution. These would be standard statistics of the process which should always be available. The

spectral density function and the bandwidth, e.g.,  $\alpha$ , should also be readily available. The general question is, "How do these parameters affect crack growth and  $S_{op}$ ?" In particular, we ask how  $\Delta K$  and R-ratio might be measured for stationary random loadings. We later discuss the treatment of nonstationary random loadings, some of the practical considerations in designing a testing program, and aspects of the calculations of the statistics of random loading fatigue crack growth.

#### 4.1 Random Loading Stress Ranges

As with constant amplitude fatigue crack growth, we should expect influences of  $\Delta K$  and R-ratio. Taking  $\Delta K$  first, a natural question is, "Which should  $da/dN$  correlate with,  $\Delta K_{mean}$  or  $\Delta K_{rms}$ ?" Note that these two parameters are actually the expected values of  $\Delta K$  raised to the powers one and two, respectively. That is,

$$\Delta K_{mean} = E[\Delta K] \quad (17)$$

and

$$\Delta K_{rms} = E[\Delta K^2]^{1/2} \quad (18)$$

Because it depends on a random process, the crack growth rate under random loading is also random. To correlate experimental data, we should probably be looking at the average or expected growth rate,  $E[da/dN]$ . Assuming the Paris fatigue crack growth law is valid, at least in first approximation, we have  $da/dN = C\Delta K^m$ . Thus, the expected value of  $da/dN$  is

$$E[da/dN] = C E[\Delta K^m] \quad (19)$$

We should thus expect  $da/dN$  to correlate with

$$\Delta K_m = E[\Delta K^m]^{1/m} \quad (20)$$

where  $m$  is dependent on the material.

Hibberd and Dover [1977] show that this gives better correlations than  $\Delta K_{mean}$  or  $\Delta K_{rms}$ . Barsom [1973] reports that the rms value works well for steels, for which it happens that  $m \approx 2$ ; this also fits the hypothesis.

The distribution of the cyclic ranges,  $\Delta K$ , must be known in order to calculate the expected value of  $\Delta K^m$ . The approximate distribution of rises/falls or the approximate distribution of rainflow cycles derived by the author [Ortiz and Chen, 1987] could be used for this distribution. These distributions are known to depend on the spectral shape, especially on the irregularity factor,  $\alpha$ .

There is still some question as to which cycle counting method is applicable to fatigue crack growth. For example, Schijve [1980] uses the rising ranges, but Sunder

et al. [1984] present fractographic evidence favoring rainflow. This is an important point that needs to be resolved.

#### 4.2 Random Loading R-Ratio

For a random loading R-ratio,  $Q = K_{mean}/K_{rms}$  and  $\gamma = S_{mean}/S_{rms}$  have been proposed and are essentially the same. One might think of  $Q$  or  $\gamma$  as measuring the number of standard deviations the mean is above zero. A large  $Q$  or  $\gamma$  (e.g., greater than 3) implies that the probability of  $X(t)$  crossing below zero is small. Hence, these ratios are indirect measures of the ratio of lowest valley to highest peak,  $R_{lh}$ .

We could propose more direct measures of the random  $R_{lh}$ . For instance, we could define

$$R_\beta = \frac{\mu_X - \beta\sigma_X}{\mu_X + \beta\sigma_X} \quad (21)$$

Assuming a Gaussian process,  $R_{\beta=1.28}$  would be, for example, the ratio of the load which  $X(t)$  is above 90% of the time and below 90% of the time. However, for the same loading this would take on different values depending on the percentage level indicated by  $\beta$ .

A more meaningful and stable ratio is the ratio of the average valley to the average peak,  $R_{\overline{min}, \overline{max}}$

$$R_{\overline{min}, \overline{max}} = \frac{E[S_{min}]}{E[S_{max}]} \quad (22)$$

If the process is Gaussian, this calculation would use the well known distributions for peaks and valleys.

Keep in mind however, that the thing we really wish to measure is the maximum plastic zone width, which we think governs the crack opening stress. Since the maximum width is caused by extreme values of the peaks and valleys, we should be looking for a statistic defined by the extremes, rather than the averages. The distinction could be significant if the distributions of peaks and valleys are highly skewed or asymmetrical. (A single parameter indicating the skewness is the ratio of the "largest" peak to the average peak.) So perhaps the ratio should be of the 90% valley to the 90% peak, for example. But, we would encounter the same difficulty regarding the percentage level as we have with  $R_\beta$  above. Furthermore, at this point in time, we do not know for what percentage level we should set the ratio.

A third approach might be taken. From the definition of the constant amplitude R-ratio,  $R = S_{min}/S_{max}$ , one can show

$$R = 1 - \frac{\Delta S}{S_{max}} \quad (23)$$

For random loadings we must substitute for  $\Delta S$  and  $S_{max}$ . Again, either the average values,  $\overline{\Delta S}$  and  $S_{\overline{max}}$ , or the  $\beta\sigma$  values, e.g., 90% values, could be used, with the same difficulties associated with  $\beta$  as before. And the question of which method to count cycles,  $\Delta S$ , needs to be resolved.

Other definitions of R could easily be proposed. A main goal of any experimental investigation should be establish which R-ratio is valid.

#### 4.3 Nonstationary Loadings

We now extend our scope to nonstationary random loadings. Many of the loadings of interest to us are nonstationary, e.g., there are ground-to-air-to-ground (GAG) cycles for aircraft and changes in vibration level for payloads during launch. This creates two new problems: the probability distributions indicated above are no longer valid, and the crack opening stress may no longer be stationary. The treatment of nonstationary processes must be on a case by case basis. However, we can address these problems in general terms.

Regarding the probability distributions, we are still interested in the distributions of cycles, peaks and valleys. However, the underlying distribution of  $X(t)$  should not be expected to be Gaussian. It is likely that the highest peaks and lowest valleys are associated with "deterministic", i.e., non-random, phenomena. For example, the lowest valley in a GAG cycle may correspond to the plane at rest on the ground, while the highest load may correspond to a certain flight maneuver. It is evident that one must study the physics of each situation in order to understand these distributions. These distributions would then be applied as before if the opening stress is still stationary.

Regarding the stationarity of the crack opening stress, it is obvious that in the occasional overloading problem the crack opening stress process is nonstationary. That is, the distribution of the process changes with time. The cause of this is the sudden change in the plastic wake width associated with the overloadings, such as illustrated in Figure 5. If overloads are extremely rare, as is the case in occasional overload testing, the analysis appears to become very complicated. However, if the overloads happen on a fairly regular basis, such as, once every flight, then the crack opening stress process might still be considered stationary, if the crack grows slowly between overloads. This is probably the case for most variable amplitude loadings of practical interest. (Besides, if the rarely occurring overload is a possibility, one probably would not want to depend on it occurring and causing retardation for a fatigue life prediction, unless the occurrence of the overload could be controlled and made to happen.)

Thus, the analysis for most nonstationary loadings would be very similar to that for stationary loadings, except for a little more effort. The question that needs

to be resolved is how slow the crack has to grow in order to maintain a smooth plastic wake.

#### 4.4 Practical Considerations for Testing

There are a number of practical considerations which must also be addressed if one is to design a testing program for random loadings. These include the truncation level and the programmed load sequence.

Theoretically, the Gaussian process  $X(t)$  has no upper or lower bounds, i.e., it may go to infinity in either direction. Naturally, this does not happen in practice and cannot be reproduced in the laboratory. In laboratory tests, the peak loadings are truncated to some finite level which is often specified by the ratio of the maximum stress to the RMS, called the clipping ratio or crest factor. It has been shown that truncating the highest loads leads to faster crack growth, due to lower opening stresses. Because of the critical impact of the largest peak on crack closure, the truncation level should always be explicitly stated. Unfortunately, many researchers do not recognize how important this is and do not state it, which makes their results ambiguous. Even though the choice of truncation level has been shown to be very important, we have no rational way of choosing it at this time. This definitely is something to be studied.

Another practical consideration has to do with programmed load sequences. In many laboratory tests, the load ranges are arranged non-randomly in a programmed sequence either from smallest to largest, or vice versa. This could have a significant impact on crack closure. The sequence of loads should be followed as faithfully as possible in a test, rather than rearranging them in such programmed sequences.

#### 4.5 Probability Calculations

Finally, regarding the calculation of the statistics of fatigue crack growth, i.e., the mean and variance of the time to failure, there are several things to note.

First, one should keep in mind that it is the residual stress behind the crack tip that controls the opening stress. So  $S_{op}$  acting for a certain load cycle is determined by loads acting some time previously. Since it is likely to take a reasonably long time for the crack to grow through the plastic zone, it is likely that the stress process acting on the crack tip would be statistically independent of the concurrent crack opening stress process. Independence of the two processes leads to certain mathematical simplifications.

A possible exception to this is for highly narrowband loadings which maintain high correlations between peak loads for long times. It is possible to imagine situations for which the concurrent stress and opening stress processes are also highly correlated (either positively or negatively). The assumption of independence should be made with care.

Second, this is a highly nonlinear problem. Therefore, the average opening stress would not necessarily lead to the average fatigue life prediction. If we want to directly calculate the average fatigue life using a constant value for  $S_{op}$ , we would need to use a value other than the expected value. This implies that we might expect large (potentially unconservative) biases from deterministic calculations of fatigue life.

Third, there is a stochastic modeling question near and dear to the author's heart. Should the opening stress process be modeled as a stochastic process evolving in time,  $S_{op}(t)$ , or in space along the crack path,  $S_{op}(a)$ ? Since the opening stress is to be mathematically manipulated with the applied stress process to get the effective  $\Delta S$ , we would prefer the time model,  $S_{op}(t)$ , for computational simplicity. On the other hand, since the opening stress is actually related to the residual stresses behind the crack tip, the space model,  $S_{op}(a)$ , might be preferred on physical grounds. This is the sort of thing professors and graduate students think about ....

## 5. CONCLUSIONS

Regarding the prediction of random loading fatigue crack growth using crack closure for fracture control analysis, we make the following observation.

- Current fracture control procedures which apply linear damage accumulation theory neglect crack closure. In this approach, the theoretically applied load spectrum is normalized so that the largest peak is equal to the maximum design load, i.e., the maximum design stress is seen once each time the spectrum is applied. Since the maximum design load should occur only rarely, perhaps never, this is thought to be "conservative", more linear damage is calculated than would be expected. On the other hand, if crack closure theory is applied, the maximum design load reduces the rate of growth by increasing the crack opening stress. Repeated application of this load would be unconservative! Veers et al. [1987] demonstrate the unconservativeness of repeated applications of such a programmed loading.

Regarding the course of future research, we make the following suggestions for theoretical and experimental work.

- There is a basic need for understanding the statistical nature of common random loadings. Are they stationary? Are they nonstationary? What are the distributions of cycles, peaks and valleys? From these statistics we would like to be able to predict whether or not crack closure is likely to have a significant impact on the fatigue life.
- The correlation of fatigue crack growth rate data with random loading equivalents for  $\Delta K$  and  $R$ , e.g.,  $\Delta K_m$  and  $R_{\overline{min}, \overline{max}}$ , should be experimentally established for stationary Gaussian processes. Different spectral bandwidths

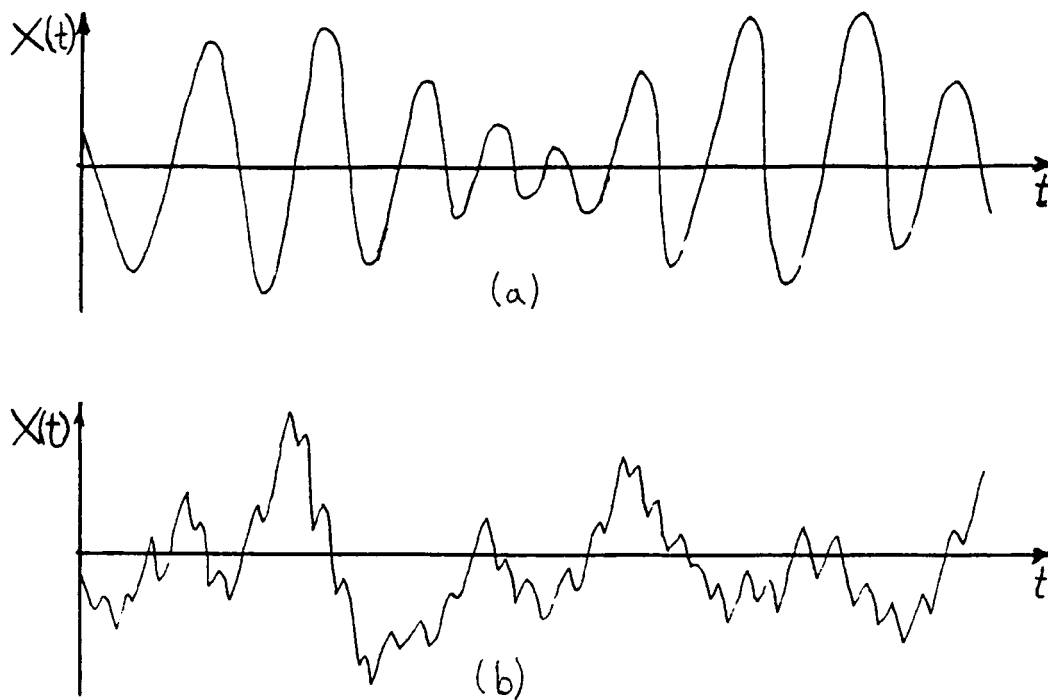


and spectral shapes should be investigated. (Beware that extra care should be taken with narrowband spectra because of the persistence of high correlations between peaks.) The loadings should be chosen so that the maximum theoretical differences in crack growth rate are calculated using the different candidates for  $\Delta K$  and  $R$ . This includes differences due to cycle counting methods.

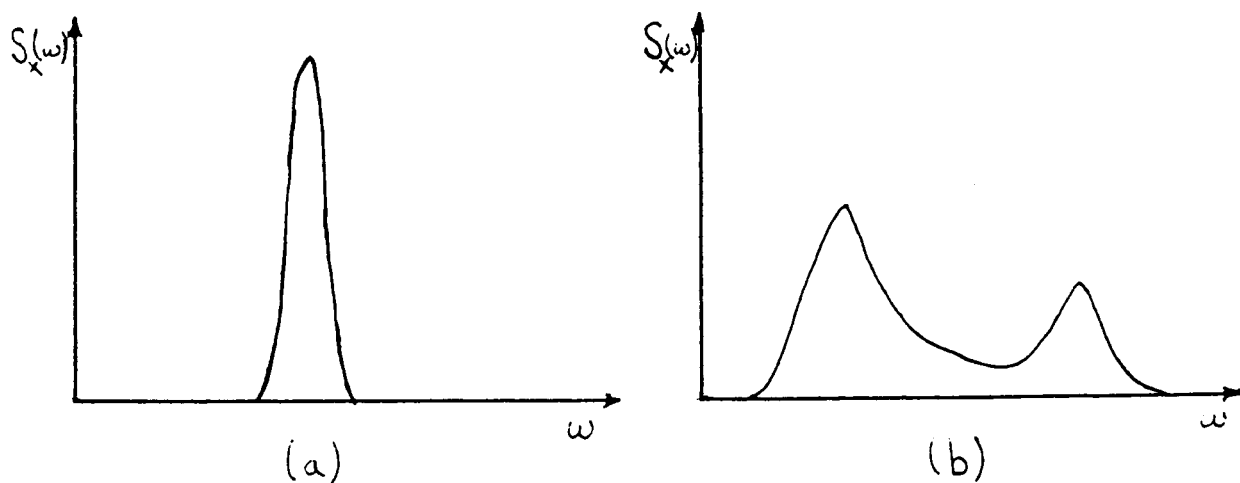
- The random loading  $\Delta K$  and  $R$ , concepts should then be extended to nonstationary random loadings. Experimental correlations should be made with the particular loading spectra of interest.
- Truncation levels should be investigated. What is the relationship between the highest peak and the second highest peak? Can general rules regarding the truncation level and the amount of retardation lost be established?
- An interesting idea which deserves exploration is the visualization of the plastic wake. It would be fascinating and informative to correlate crack growth rate with the structure of the wake, for instance as depicted in Figure 5. How might the wake be seen? Photographs against grids have been used to show gross plastic deformation. Some sort of surface treatment or etching might show the actual extent of plastically deformed material in finer detail.
- Finally, the impact of adopting crack closure models into fatigue crack growth analysis for fracture control should be investigated. Using a more accurate crack growth analysis for random loadings places a greater burden on inspection to screen initial flaws. The confidence in the inspection should be greater. How much greater? An effort should be made to quantify the probability of fracture using current fracture control practices and using crack closure. The theoretical probability of fracture should be kept constant, unless a change can be justified.

## 6. REFERENCES

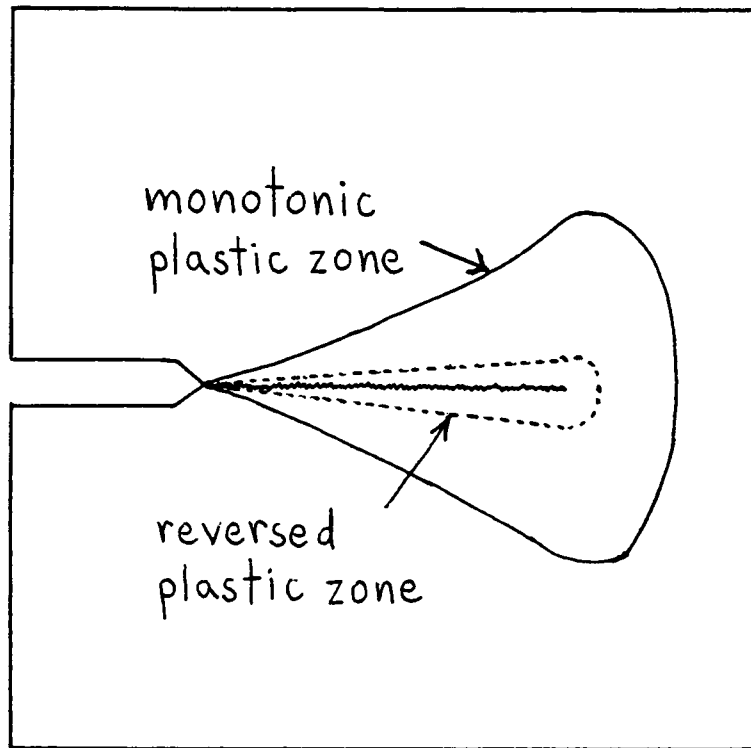
- Banerjee, S., "A Review of Crack Closure," Materials Laboratory, Air Force Wright Aeronautical Laboratories, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, AFWAL-TR-84-4031, April 1984.
- Barsom, J.M., ASTM STP 536, 1973, pp. 147-167.
- Bendat, Julius S., and Piersol, Allan G., *Random Data, Analysis and Measurement Procedures*, John Wiley & Sons, 1986.
- Elber, W., in *Damage Tolerance in Aircraft Structures*, ASTM STP 486, American Society for Testing and Materials, 1971, pp. 230-242.
- Forman, R.G., et al., "Fatigue Crack Growth Computer Program NASA/FLAGRO," Materials Branch, Structures and Mechanics Division, NASA Lyndon B. Johnson Space Center, Houston, Texas, JSC-22267, August 1986.
- Hibberd, R.D., and Dover, W.D., "The Analysis of Random Load Fatigue Crack Propagation," *Fracture 1977, Fourth International Conference on Fracture*, Waterloo, Canada, ICF4, Vol. 2, June 1977, pp. 1187-1194.
- Newman, J.C., Jr., "A Crack Opening Stress Equation for Fatigue Crack Growth," *International Journal of Fracture*, Vol. 24, 1984, pp. R131-R135.
- Newman, J.C., Jr., personal communication with R.G. Forman on May 20, 1987, unpublished.
- Ortiz, Keith, and Chen, Nobel K., "Fatigue Damage Prediction for Stationary Wide-band Random Stresses," *ICASP5, Fifth International Conference on Applications of Statistics and Probability in Soil and Structural Engineering*, Vancouver, Canada, May 1987.
- Schijve, Jaap, "Prediction Methods for Fatigue Crack Growth in Aircraft Material," *Fracture Mechanics: Twelfth Conference*, ASTM STP 700, American Society for Testing and Materials, 1980, pp. 3-34.
- Sunder, R., Seetharam, S.A., and Bhaskaram, T.A., "Cycle Counting for Fatigue Crack Growth Analysis," *International Journal of Fatigue*, Vol. 6 No. 3, 1984, pp. 147-156.
- Veers, Paul S., Winterstein, Steven R., Nelson, Drew V., and Cornell, C. Allin, "Variable Amplitude Load Models for Fatigue Damage and Crack Growth," submitted for review to, *ASTM Symposium on Fatigue Loading Spectra*, 1987.



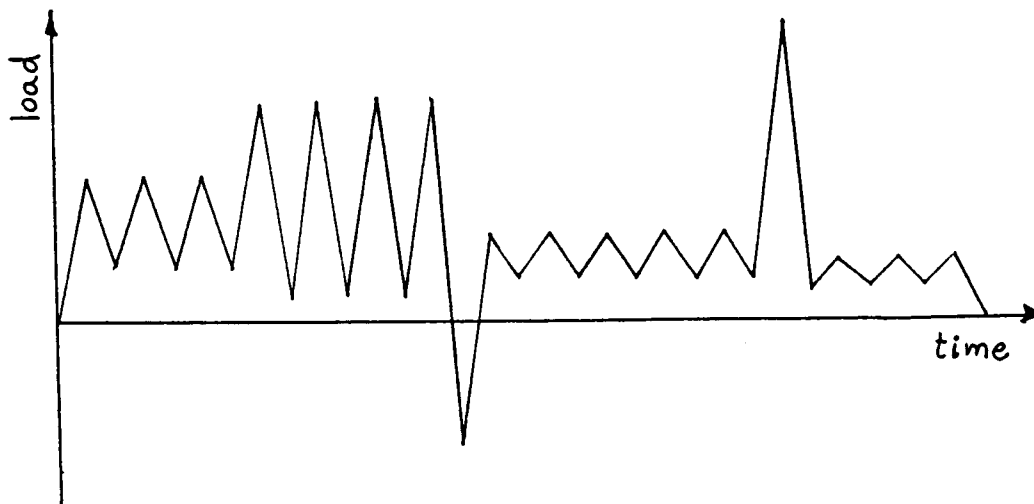
**Figure 1.** Sample random processes: (a) narrowband; (b) wideband.



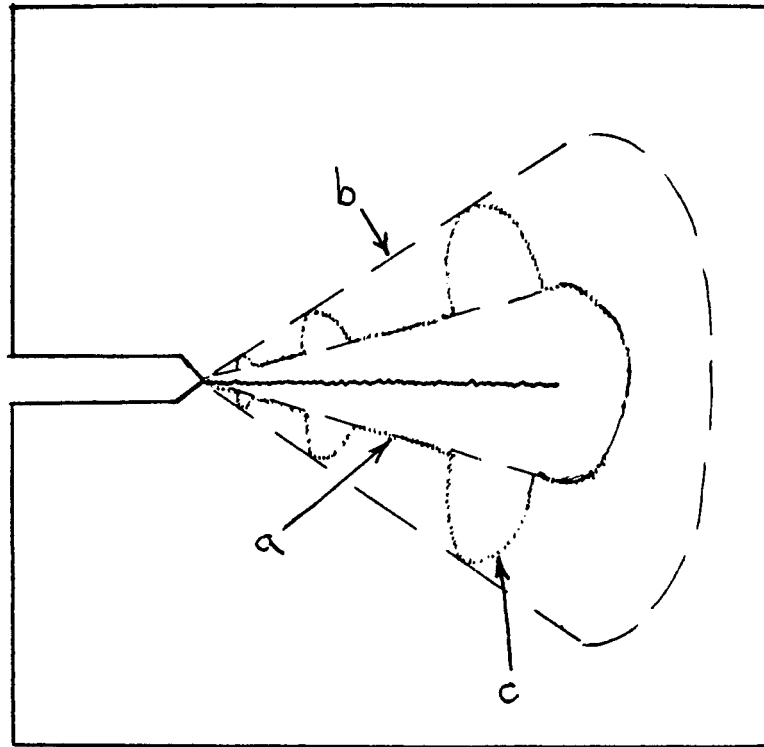
**Figure 2.** Spectral density functions (one-sided) for processes in Figure 1: (a) narrowband; (b) wideband.



**Figure 3.** Illustration of the plastic wake for constant amplitude loading showing the envelope of monotonic plastic zone and the reversed plastic zone.



**Figure 4.** Example variable amplitude fatigue loading.



**Figure 5.** Illustration of the plastic wake for: (a) constant amplitude loading from 0 to  $S_1$ ; (b) constant amplitude loading from 0 to  $S_2$  greater than  $S_1$ ; (c) constant amplitude loading from 0 to  $S_1$  with occasional overloads to  $S_2$ .